CS225 Assignment 1

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1. a.

p	q	$\neg q$	$p \to \neg q$	$(p \to \neg q) \vee q$	$((p \to \neg q) \vee q) \to p$
0	0	1	1	1	0
0	1	0	1	1	0
1	0	1	0	0	1
1	1	0	1	1	1

Contingent

b.

p	q	$\neg q$	$\neg q \vee p$	$(\neg q \vee p) \wedge q$	$((\neg q \vee p) \wedge q) \to p$
0	0	1	1	0	1
0	1	0	0	0	1
1	0	1	1	0	1
1	1	0	1	1	1

Tautology

2. a.

p	q	r	$p \vee q$	$\neg(p \vee q)$	$\neg(p \vee q) \wedge r$	$\neg q$	$(\neg(p \vee q) \wedge r) \to \neg q$
0	0	0	0	1	0	1	1
0	0	1	0	1	1	1	1
0	1	0	1	0	0	0	1
0	1	1	1	0	0	0	1
1	0	0	1	0	0	1	1
1	0	1	1	0	0	1	1
1	1	0	1	0	0	0	1
1	1	1	1	0	0	0	1

Tautology

b. We want to show that $(p \to q) \wedge (q \vee \neg r) \Leftrightarrow (p \vee r) \to q$:

$(p \to q) \wedge (q \vee \neg r) \equiv (\neg p \vee q) \wedge (q \vee \neg r)$ *Implication*

$\equiv (\neg p \wedge \neg r) \vee q$ *Distribution/Factoring*

$\equiv \neg(p \vee r) \vee q$ *De Morgans*

$\equiv (p \vee r) \to q$ *Implication*

Therefore, $(p \to q) \wedge (q \vee \neg r) \Leftrightarrow (p \vee r) \to q$

3\. a. $((\neg p \to q) \land q) \to \neg p$ is not a tautology. Let $p = T$ and $q = T$. This gives us:

$$((\neg T \to T) \land T) \to \neg T \equiv ((F \to T) \land T) \to F$$
$$\equiv (T \land T) \to F$$
$$\equiv T \to F$$
$$\equiv F$$

Therefore, $((\neg p \to q) \land q) \to \neg p$ is not a tautology.

b. $((p \lor \neg q) \land p) \to p$ is a tautology. We can see this by:

$$((p \lor \neg q) \land p) \to p \equiv ((p \land p) \lor (p \land \neg q)) \to p \quad \text{*Distribution*}$$
$$\equiv (p \lor (p \land \neg q)) \to p \quad \text{*Idempotent*}$$
$$\equiv p \to p \quad \boxed{\text{Tautology}}$$

Therefore, $((p \lor \neg q) \land p) \to p$ is a tautology.

4\. a. Given that n is an integer, we want to show that if $2n^2 + 3n$ is odd then n is odd. We will do this using contraposition, showing that if n is not odd, then $2n^2 + 3n$ is not odd. Because we know n is an integer, we can also write this as if n is even then $2n^2 + 3n$ is even.

Let $n = 2k$

Then $2n^2 + 3n = 2(2k)^2 + 3(2k) = 2(4k^2) + 6k = 2(4k^2) + 2(3k)$

$4k^2, 3k \in \mathbb{Z}$, therefore $2(4k^2)$ and $2(3k)$ are both even by definition. We also know that the sum of two even integers will be even, giving us $2(4k^2) + 2(3k)$ is even.

We can then conclude that if n is even then $2n^2 + 3n$ is even, and, by contraposition, if $2n^2 + 3n$ is odd, then n is odd.

b. We want to show that the converse of the statement, "If $2n^2+3n$ is odd, then n is odd", is also true. This can be written as, "If n is odd, then $2n^2+3n$ is odd."
Let $n=2k+1$, where $k \in \mathbb{Z}$.
Then $2n^2+3n = 2(2k+1)^2+3(2k+1) = 2(4k^2+4k+1)+6k+3$

$$= 8k^2+8k+2+6k+3 = 8k^2+14k+5$$

$$= 2(4k^2+7k+2)+1$$

Since k is an integer we know that $4k^2+7k+2$ is an integer by integer multiplication and addition.
Then let $4k^2+7k+2=j$ and we can say $j \in \mathbb{Z}$.
We can then re-write $2(4k^2+7k+2)+1$ as $2(j)+1$ which is odd by definition. Therefore, by direct proof we can say that if n is odd, then $2n^2+3n$ is odd.

5. a. To prove that if $n^2+n>0$, then $n \notin \{-1,0\}$ we will use proof by contraposition and cases. The contrapositive of the statement can be written as, "If $n \in \{-1,0\}$, then $n^2+n \leq 0$."
Take the case where $n=-1$:

$$(-1)^2+(-1) \leq 0 \Rightarrow 1 - 1 \leq 0 \Rightarrow 0 \leq 0 \quad \text{Which is true}$$

Then take the case where $n=0$:

$$(0)^2+(0) \leq 0 \Rightarrow 0+0 \leq 0 \Rightarrow 0 \leq 0 \quad \text{Which is true}$$

Since all cases hold true, we can say that if $n \in \{-1,0\}$, then $n^2+n \leq 0$, and by contraposition, if $n^2+n>0$, then $n \notin \{-1,0\}$.

b. The converse of this statement, which can be written as, "If $n \notin \{-1,0\}$, then $n^2+n>0$", is also true.

We will show this with a proof by cases.

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Since n is an integer and $n \in \{-1, 0\}$ we have two possibilities. Either $n > 0$ or $n < -1$. We can also say that $n^2 + n = n(n+1)$.

Take the case where $n > 0$. In this case n is positive and therefore $n+1$ is also positive. This makes the term $n(n+1)$ a product of two positive numbers greater than 0 which makes $n(n+1)$ a positive number greater than 0 by definition.

Next, take the case where $n < -1$. In this case n is a negative number less than -1 so we know that $n+1$ is also negative and must be less than 0. This makes the term $n(n+1)$ a product of two negative numbers less than 0 which makes $n(n+1)$ a positive number greater than 0 by definition.

Since all cases hold true we can say by proof by cases that if $n \notin \{-1, 0\}$, then $n^2 + n > 0$.